AERA 2016 Symposium Session:
Discovery-Based STEM Learning 2.0: Are We There Yet?

Chair: Dor Abrahamson
Organizer: Kiera Chase
Discussant: Douglas Clements

Abstract
As we celebrate the centennial of Dewey's 'Democracy and Education,' we soberly appraise whether our field has realized his vision of meaningful situated learning, and we look forward to imagine the journey that lies ahead. Session presenters will each situate their contribution in a design study of STEM cognition and instruction. Each group arrived at a different design solution for achieving organization and flow of learning activities (i.e., the task, resources, and facilitation) that optimize the balance of under- and over-constraining students' discovery process. We thus present a scope of design heuristics that may generalize across STEM domains. Our discussion will remain grounded in the theory of learning and its ongoing evaluation, refinement, and expansion through design-based research empirical studies.

Session Summary
A century after the first publication of Dewey's *Democracy and Education* (1916) we appraise whether the field of educational research has lived up to his vision.

Dewey advocated for instructional practices that would offer students engaging, meaningful, and personally relevant experiences—what we might now call situated learning. His view resonates with complementary pedagogical frameworks, and in particular those shaped by the epistemological theory of Jean Piaget (1947/1963), such as constructivism (e.g., Kamii & DeClark, 1985), radical-constructivism (von Glasersfeld, 1983), and the maker-oriented corollary, constructionism (Papert, 1980). Consequently, a plethora of Deweyan/Piagetian instructional-design frameworks—alternatively problem-, project-, inquiry-, apprentice-, modeling-, or discovery-based—have been implemented in STEM classrooms, leading to curricular shifts toward more open-ended activities. In mathematics, one example is the problem-based school, Realistic Mathematics Education (Freudenthal, 1968; Gravemeijer, 1994); in science, inquiry-based activities have served to scaffold science methodology (Quintana et al., 2004). In parallel, technological advances, such as new HCI platforms, have enabled better to cultivate young people's agency and empowerment by having them engage in authentic or simulated STEM practice. While US STEM instruction still largely runs in tell-and-practice sequences (Schwartz et al., 2011), reversing this canonical sequence makes for more engagement, deeper learning, and longer retention (Chase & Abrahamson, in press; Holmes et al., 2014; Kapur, 2014; Levy, 2012; Schneider et al., in press; Wilkerson-Jerde et al., 2015b).

Notwithstanding, the validity and effectiveness of these approaches has been called in question, with some of the recurring plaints being that minimal guidance results in students generating faulty ideas that are hard to override, the instructional process is time consuming, students do not have opportunities to practice and elaborate on solution strategies, and transferring knowledge to new situations can is compromised (Kirschner et al., 2006; Klahr & Nigam, 2004). And whereas these critiques have been addressed eloquently (Nathan, 2012), battles continue even as progressive Standards settle in. It appears that further research is called for, both to unpack these nettled issues and to set examples for effective reform-oriented instruction.

This symposium will bring together a set of responses to critiques leveled against discovery-based pedagogy. Common to these contributions is that each will present an empirical evaluation of an instructional design. Each paper demonstrates a unique use of media to implement activities that enable the situated construction of new ideas, offer opportunities to practice new skills and elaborate insight in new problem contexts, and throughout provide appropriate guidance and support.

The contributions differ with respect to their location along a hypothetical axis running between under- and over-constraining the discovery process. Furthermore, the contributions present designs implemented in a variety of contexts (content, setting, age), technological media (concrete, virtual, remote), and assessment climates (e.g., mastery).

Following 5 minutes of introductory comments from the symposium Chair, 6 papers will each be presented over 13 minutes. The session will end with a 20-minute commentary from our Discussant, Dr. Douglas Clements, an international expert on reform-oriented uses of technology for educational design and their consequences for student learning and school adoption.

Searching For Buried Treasure: Uncovering the Discovery in Discovery-Based Learning

Kiera Chase & Dor Abrahamson

Objective

What's discovered in discovery-based learning? In the case of motor-action skills, Vereijken and Whiting (1990) write, "Discovery learning forces the learner to explore the dynamics of the system in which he or she operates" (p. 99). What might that possibly look like in mathematics, specifically algebra? For algebra, discovery learning might be tantamount to revealing-systemic relations, structures, and functions for the handling of propositions toward determining unknown values. Yet what are these basic elements of algebra, and how might a student discover them? Perhaps the best way to understand a system is to build it (Goldstein & Papert, 1977)! But then which resources, tasks, feedback, and facilitation might optimize this reinvention of algebra? As a means of creating empirical context to investigate discovery learning, we developed a learning environment, Giant Steps for Algebra (GS4A).

Theoretical Approach

We conceptualize discovery-based learning as students making *transparent* for themselves how cultural artifacts function to mediate the achievement of collective objectives (Hancock, 1995; Meira, 1998). Where artifacts are epistemic, such as in mathematics, developing subjective transparency of the artifact *is* learning the content.

Methods

Instructional methodology has yet to enable students to build cognitive bridges from arithmetic to algebra (Herscovics & Linchevski, 1994; Molina & Ambrose, 2008).

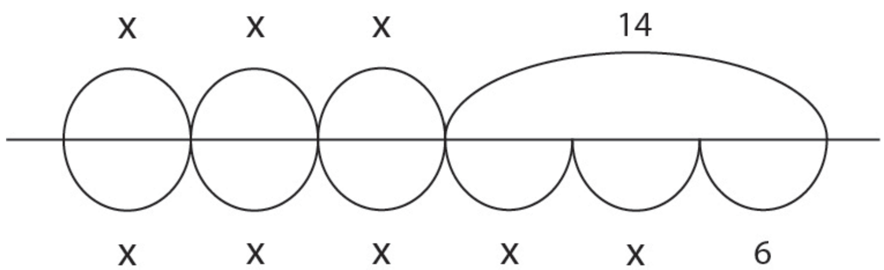

Figure 1. Dickinson and Eade's (2004) Number-line instantiation of "3*x* + 14 = 5*x* + 6"

A Giant walked 3 steps and then another 2 meters. She buried the treasure. On the next day, she wanted to bury more treasure in exactly the same place, but she was not sure where that place was. She walked 4 steps and then, feeling she'd gone too far, she walked back one meter. Yes! She found the treasure!

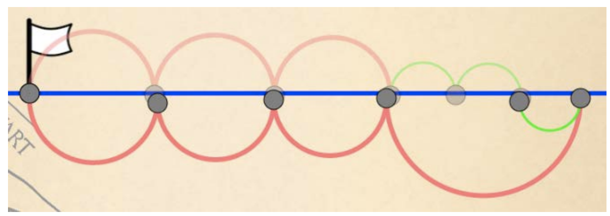

Figure 2. GS4A narrative and model. On Day 1, above the line, and Day 2, below the line, the giant travels from the flag. Red loops represent giant steps, green loops represent meters.

Our design sought to render algebraic content transparent by:

a. selecting a model that enhances potential saliences of relations among variable and integers (see Figure 1);
b. engaging students' tacit knowledge of quantitative relations latent to simple motion in space (Abrahamson, 2009, 2014);
c. leveraging student narrative practices by posing enactment problems (Walkington, et al., 2013); see Figure 2);
d. assigning modeling tasks: students enact the narratives by utilizing available objects and functions, then reflect on embedded systemic features of their models; and
e. implementing the design in a digital environment (Sarama & Clements, 2009).

A pilot study (Abrahamson & Chase, 2015) implicated three transparency goals emerging from student reflection on their own models' embedded structures: (1) consistent measures; (2) equivalent expressions; and (3) shared frame of reference. We conceptualized these goals as "situated intermediary learning objectives" (SILOs):

Our experimental design compared learning achievement under two conditions:
1. Discovery: each SILO functionality is automated only after the user discovers it
2. No-Discovery: all three SILO functionalities are always automated

Forty 4th and 9th Grade students, randomly assigned to experimental condition, voluntarily participated individually in task-based semi-structured interviews (Clement, 2000; Ginsburg, 1997). Mixed methods were applied, including post assessment and micro-genetic analyses.

Results
"Discovery" participants significantly outperformed "No-Discovery" participants in attaining the SILOs. Moreover, functionalities that "No Discovery" participants conceptualized as mere features, "Discovery" participants conceptualized as essential procedural functions.

Significance
Students learn mathematical concepts by making transparent structures and functions embedded in their spontaneous solution procedures. These discoveries are situated intermediary learning objectives that the activity elicits as the meaning underlying prospective formal procedures.

How Exploratory Problem-Solving Versus Problem-Posing Help Learning From instruction

Manu Kapur
National Institute of Education (Singapore)

Objective
There is now a growing body of evidence that preparatory activities such as generating solutions to novel problems *prior* to instruction can help students learn better from the instruction (Kapur & Rummel, 2012). Evidence comes not only from quasi-experimental studies conducted in the real ecologies of classrooms (e.g., Kapur, 2012, 2013; Schwartz & Bransford, 1998; Schwartz & Martin, 2004), but also from controlled experimental studies (e.g., DeCaro & Rittle-Johnson, 2012; Kapur, 2014; Loibl & Rummel, 2013, 2014; Roll, Aleven, & Koedinger, 2011; Schmidt & Bjork, 1992; Schwartz, Chase, Oppezzo, & Chin, 2011).

The preparatory activities in above studies present students with contexts where the problem is given. However, an equally, if not more, important mathematical skill is to generate problems in the first place (Jay & Perkins, 1997; Silver, 1994). The proposition being: students need to be provided opportunities for both problem-solving and problem-posing.

In two randomized-controlled studies, I compared the preparatory effects of problem-solving versus problem-posing on learning from subsequent instruction.

FOOTBALL STRIKERS

Mike and Dave are the top two strikers in a Football league. The table shows the number of goals scored by Mike and Dave over the course of 11 games in the league.

An award has to be given to the more consistent player of the two. The decision has to be made mathematically.

Goals scored by Mike and Dave

Game	Mike	Dave
1	14	13
2	11	11
3	15	14
4	12	16
5	16	14
6	12	12
7	16	14
8	13	15
9	17	14
10	14	17
11	14	14

Generate as many measures of consistency as you can to determine the more consistent player. Show all working.

Figure 3. Problem-solving before instruction

FOOTBALL STRIKERS

Mike and Dave are the top two strikers in a Football league. The table shows the number of goals scored by Mike and Dave over the course of 11 games in the league.

Generate as many different mathematics questions or problems that can be answered from the information provided in the table.

Goals scored by Mike and Dave

Game	Mike	Dave
1	14	13
2	11	11
3	15	14
4	12	16
5	16	14
6	12	12
7	16	14
8	13	15
9	17	14
10	14	17
11	14	14

Where possible, answer or solve the problems/questions you have generated. Show all working.

Figure 4. Problem-posing where students generate problems and solutions before instruction

Methods
In Study 1, students engaged in either problem-solving (where they generated solutions to a novel problem; see Fig 3) or problem-posing (where they generated problems, and where possible, the associated solutions; see Fig. 4) prior to learning the math concept of Standard Deviation. Study 1 found that problem-posing prior to instruction resulted in significantly better transfer to novel problems than problem-solving, without any significant difference in procedural knowledge and conceptual understanding.

FOOTBALL STRIKERS

Mike and Dave are the top two strikers in a Football league. The table shows the number of goals scored by Mike and Dave over the course of 11 games in the league.

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6	12	12
7	16	14
8	13	15
9	17	14
10	14	17
11	14	14

You do NOT need to answer or solve the problems/questions you have generated. Just generate as many questions as possible.

Figure 5. Problem-posing without needing to solve the problems students generate.

Study 2 was designed to further examine the trade-off between problem and solution generation on conceptual understanding and transfer. Students engaged in either problem-solving (as in Fig 3) or problem-posing (where they generated only problems without solutions; see Fig. 5) prior to learning the novel math concept.

Results
Findings showed that problem-solving prior to instruction resulted in better conceptual understanding than problem-posing without solutions. However, the transfer effect remained in favor of problem-posing, albeit weaker than in Study 1.

Significance
Taken together, Study 1 suggested a trade-off between the benefits of wider activation afforded by problem-posing and that of a more relevant activation afforded by problem-solving. Study 2 further demonstrated that absent the opportunity to generate solutions, conceptual understanding suffered, and that the wider activation afforded by problem generation was not sufficient to compensate for the relevant activation due to solution generation.

The transfer findings are intriguing. In both studies, the problem-posing afforded wider knowledge activation, even though not all relevant to the targeted concept. Furthermore, the number of problems generated was the most strongly correlated with transfer performance. Even though this correlation was relatively weaker in Study 2 (problem posing with only problem generation) than in Study 1 (problem posing with problem and solution generation), it was still stronger than the same for solution generation. These findings suggest that whereas solution generation and its attendant preparatory mechanisms play an important role for transfer (given the correlation between solution generation and transfer), problem generation, even if part of it is not directly relevant, plays a more critical role.

Scaffolding is a Double-Edged Sword: Looking Into Individual Differences in Attitudes and Knowledge

Ido Roll, Nikki Yee, Deb Butler, Joss Ives, Georg Rieger, Doug Bonn, & Ashley Welsh
University of British Columbia

Objective

Discovery-based learning is often criticized for letting students flounder in the absence of sufficient guidance (Alfieri, Brooks, Aldrich, & Tenenbaum, 2011). However, prior studies typically: (a) examined only knowledge acquisition; and (b) aggregated results across all learners. To better understand the effect of guidance on learning, we asked: How does scaffolding interact with prior knowledge and attitudes in the context of discovery-based learning?

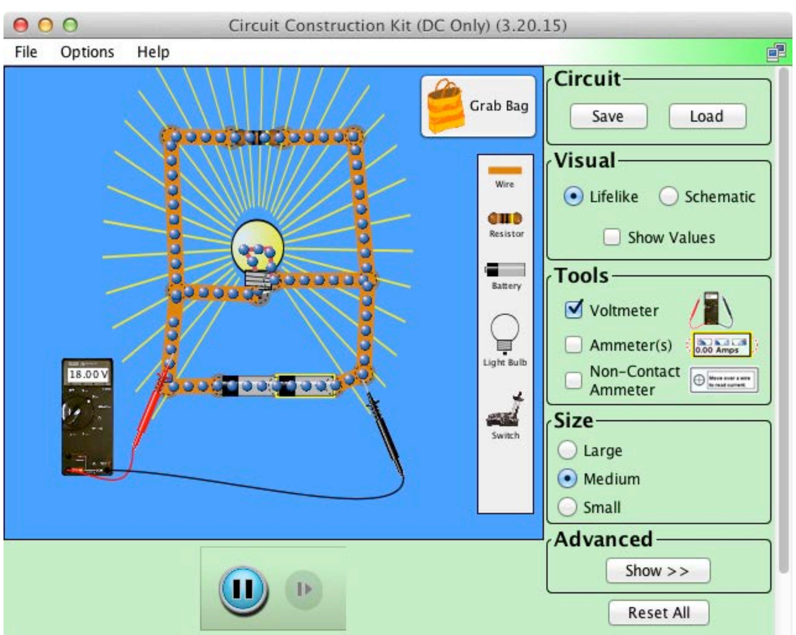
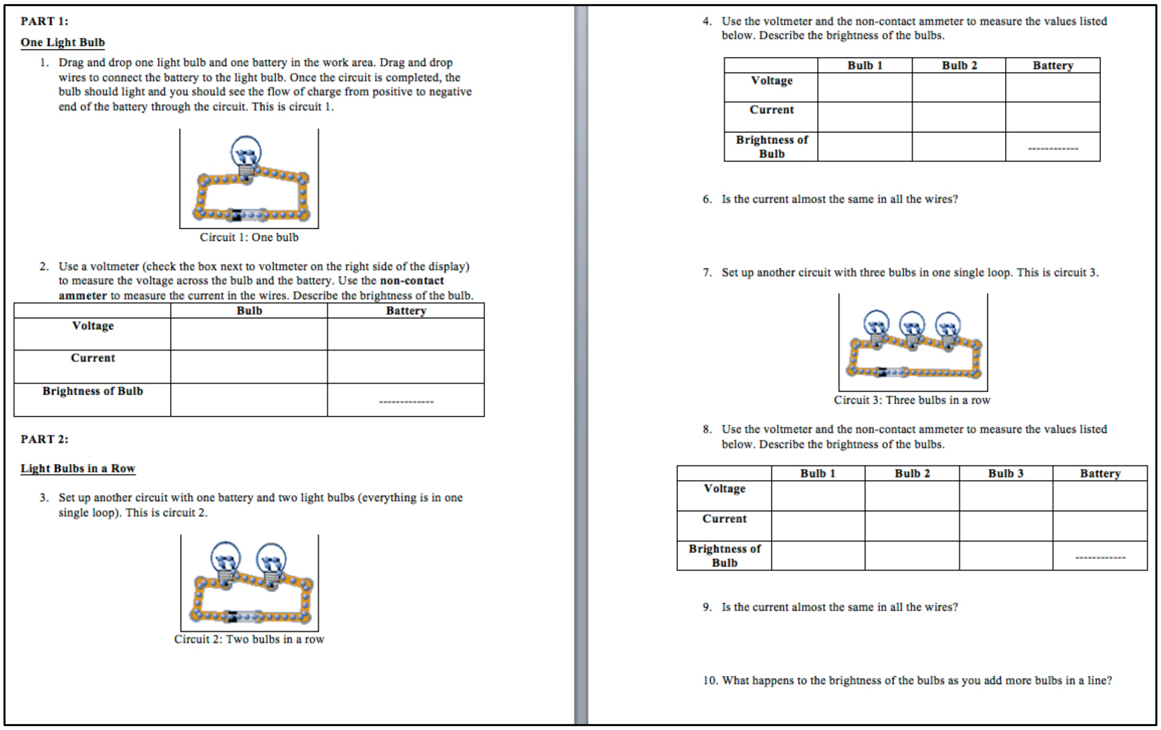


Figure 6: The PhET Circuit Construction Kit (CCK). The simulation gives immediate feedback by animating light intensity and electron speed.



PART 1:

One Light Bulb

1. Drag and drop one light bulb and one battery in the work area. Drag and drop wires to connect the battery to the light bulb. Once the circuit is completed, the bulb should light and you should see the flow of charge from positive to negative end of the battery through the circuit. This is circuit 1.

Circuit 1: One bulb

2. Use a voltmeter (check the box next to voltmeter on the right side of the display) to measure the voltage across the bulb and the battery. Use the **non-contact ammeter** to measure the current in the wires. Describe the brightness of the bulb.

	Bulb	Battery
Voltage		
Current		
Brightness of Bulb		------------

PART 2:

Light Bulbs in a Row

3. Set up another circuit with one battery and two light bulbs (everything is in one single loop). This is circuit 2.

Circuit 2: Two bulbs in a row

4. Use the voltmeter and the non-contact ammeter to measure the values listed below. Describe the brightness of the bulbs.

	Bulb 1	Bulb 2	Battery
Voltage			
Current			
Brightness of Bulb			-----------

6. Is the current almost the same in all the wires?

7. Set up another circuit with three bulbs in one single loop. This is circuit 3.

Circuit 3: Three bulbs in a row

8. Use the voltmeter and the non-contact ammeter to measure the values listed below. Describe the brightness of the bulbs.

	Bulb 1	Bulb 2	Bulb 3	Battery
Voltage				
Current				
Brightness of Bulb				-----------

9. Is the current almost the same in all the wires?

10. What happens to the brightness of the bulbs as you add more bulbs in a line?

Figure 7. Sample worksheet in the Scaffolded condition

Methods

The study evaluated two extreme forms of scaffolding. 97 college students participated either in a *scaffolded* (48) or *unscaffolded* (49) learning activity using a PhET Circuit Construction Kit (CCK; https://phet.colorado.edu/en/simulation/circuit-construction-kit-dc) for simulating D/C circuits (Figure 6).

All participants explored over 25 minutes how light bulbs' voltage, current, and brightness depend on their number and arrangement. Only Scaffolded participants received compare-and-contrast scenarios and reflection prompts that had been co-designed in collaboration with the PhET team and course instructor (Figure 7).

Domain knowledge was assessed using conceptual pre- and post-tests. Attitudes were assessed using pre- and post-surveys, modeled after Butler, Cartier, Schnellert, Gagnon, and Giammarino (2011). Surveys included ten 4-point Likert scale items, evaluating Perceptions of Competence and Control (PoCC). The single PoCC scale used here includes the average of 9 of items, Cronbach α=0.82. The pre-survey also surveyed goal orientation and perceived value of working with PhET simulations.

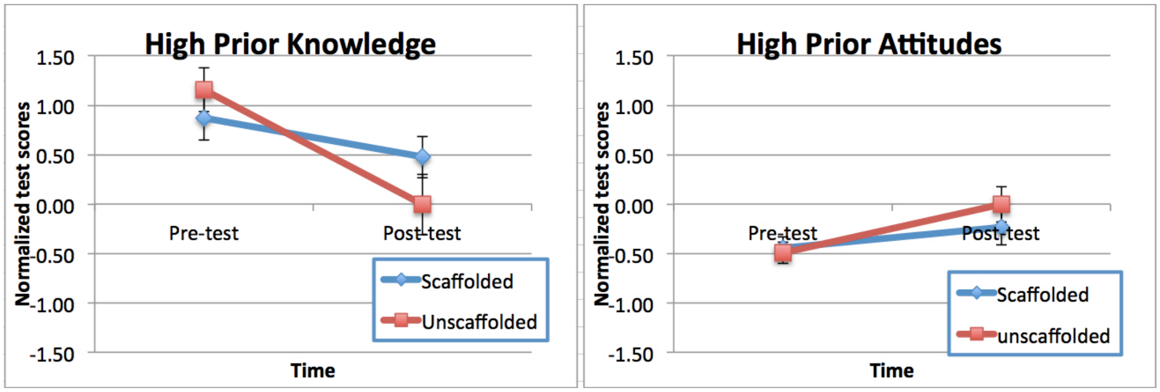


Figure 8. Effect of scaffolding on knowledge in both groups

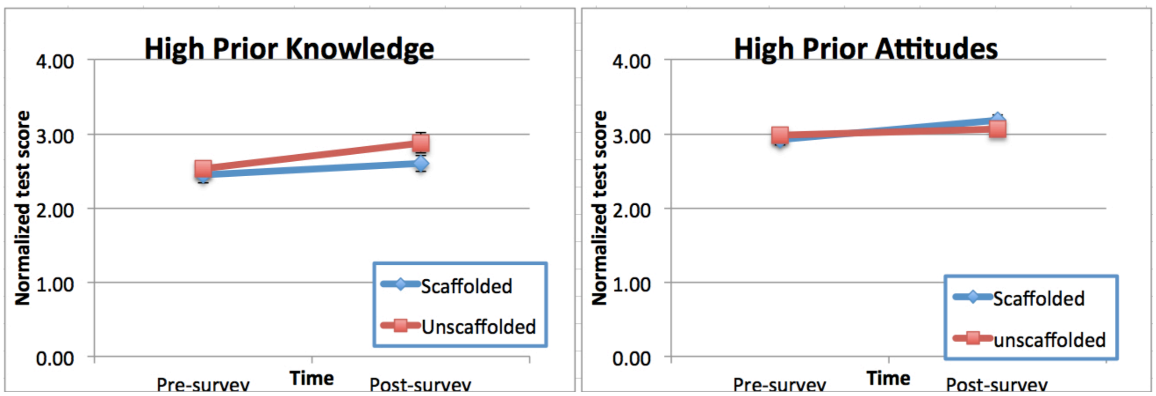


Figure 9. Effect of scaffolding on attitudes in both groups

Results
Pre-intervention findings. Independent of their study condition, we clustered participants based on prior-knowledge, prior-PoCC, perceived value of using PhET, and Mastery orientation. Intriguingly, our analysis revealed two statistically differentiated clusters of learners: 33 students came in with high prior knowledge but lower "attitudes" (High-Knowledge group). 61 students had lower pre-test scores but were more enthusiastic and confident about learning with PhET (High-Attitudes group).

All participants improved significantly from pre (0.47±0.17) to post (0.62±0.23); t(96)=6.1, p<0.0005. Figure 8 shows the impact of scaffolding on normalized test-scores. ANCOVA of post-test as a function of Condition and Cluster, controlling for pre-test and pre-survey, shows significant Condition*Cluster interaction: F(1,91)=3.83, p=0.05. While scaffolding assisted students in the High-Knowledge group, it hurt students in the High-Attitudes group.

Figure 9 shows the impact of scaffolding on attitudes. A similar ANOCVA with PoCC as a dependent measure found a significant Condition*Cluster interaction: F(1,91)=5.2,

p=0.025. Whereas study condition did not affect the attitudes of the High Attitudes group, it is the no-scaffolding treatment that brought High-Knowledge students to the same level as their High-Attitudes counterparts.

In sum, only students with sufficient prior-knowledge benefited from scaffolding, likely because compare-and-contrast processes require background knowledge in order to be beneficial. Low-prior-knowledge students benefited more from following their own trajectories in the *Unscaffolded* condition (cf. Roll, Baker, Aleven, & Koedinger, 2014). Also, using scaffolding to direct students' learning negatively affected students' self-perceptions of competence and control.

Significance
It is possible that the quest for highly-efficient learning of specific topics in science comes at the expense of promoting authentic inquiry, which is messier, more exploratory, and prone to errors by its very nature.

TrafficJams: Collaborative Exploration of Driving and Traffic

Sharona T. Levy, Ran Peleg, Eyal Ofeck, Ilana Dubovi, Naamit Tabor, Shiri Bluestein, & Hadar Ben-Zur
University of Haifa

Theoretical Approach
Discovery learning proponents designs that support active knowledge construction (Bruner, 1961) including open-ended problem-based, inquiry-based and design-based learning, showing advantages to learning and especially its transfer (Kapur, 2008). Opponents demonstrate how unguided discovery is ineffective, based on cognitive load and expertise (Mayer, 2004; Kirchner et al., 2006).

Objective
This preliminary design-experiment approaches this quandary by investigating learning with a participatory-simulation (Resnick & Wilensky, 1998; Colella, 2000) that supports open-ended *collaborative endeavors* within *responsive constrained* environments, by planning, enacting and interpreting social experiments. Collaborative endeavors, distinct from design or inquiry, widen the scope of activities that can take place. Constraints limit the range of possible actions. Responsiveness relates to feedback to effected actions. Pairing of environmental responses to constrained action provides for discovery of underlying rules.

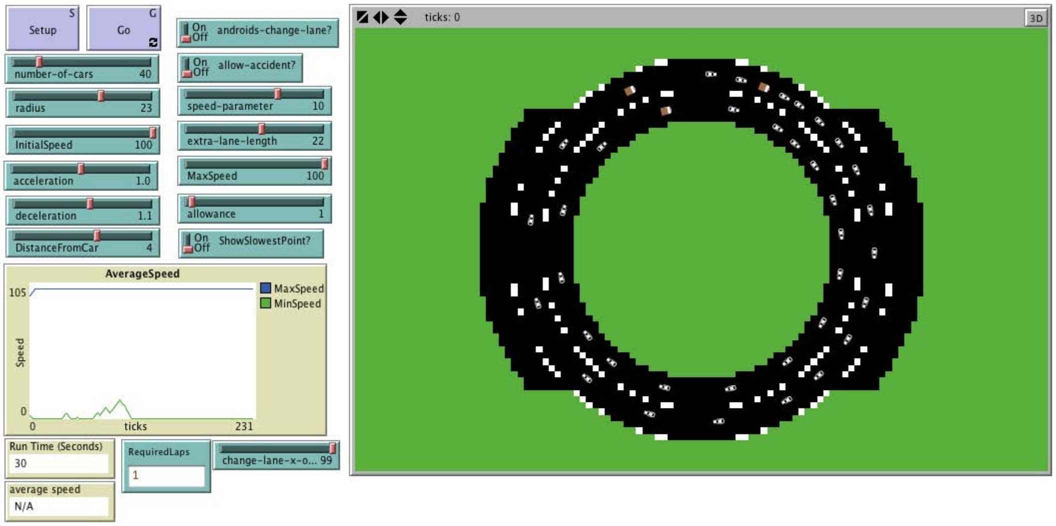


Figure 10. TrafficJams participatory simulation interface

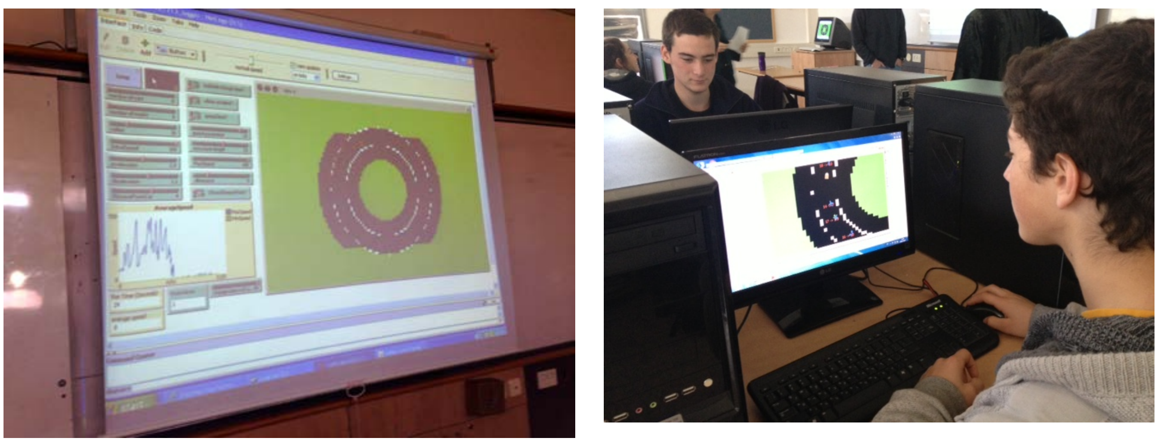


Figure 11. Classroom setup for TrafficJams simulation activity, including the projected class simulation (left) and students' individual clients (right).

Methods

Design involved analysis of traffic, its paradoxes and misunderstandings by drivers (Kerner, 2004), resulting in goals: making such paradoxes apparent in the system's responses and supporting enactment of a wide range of endeavors. Participants use the TrafficJams simulation (Levy et al., 2014, see Figure 10 & 11) driving collaboratively on a two-lane road (constraints) and in the process may discover the relationships between driving and traffic. TrafficJams was created with NetLogo HubNet (Wilensky & Stroup, 1999).

Learning of one high-school class was studied with a pretest-intervention-posttest design during 2.5 hours. 29 students participated, 16 completed both tests. Identical pre- and post-assessments ("self-aggressive driving", Sikron, Baron-Epel & Linn, 2007; locally-created "understanding traffic"), observations and data-logging.

Table 1: Traffic understanding questionnaire results, in terms of changes overall and individual concepts (frequency out of N=16).

Dimension	**Improvement**	**Regression**	**Same**
Overall	9	2	5
Decentralized view	4	0	12
Multiplicity of causes for jams	3	1	12
Car speed distribution as generating jams	3	0	13
Car speed variation as generating jams	1	0	15
Bottom-up reasoning	2	0	14
Top-down reasoning	0	1	15
Mechanistic reasoning	2	2	11

Results

After free-driving to gain experience, the first challenge was making traffic jam. Students proposed three strategies, voted and two were enacted: driving closely and frequent lane changes. They chose to include accidents, which resulted in much gore and joy… Smoothing traffic was the next challenge and students designed two strategies: All cars in one lane, and having everybody drive at the same speed. Discussions included how an individual could impact the collective and vice versa, frustration at not being first overriding willingness to cooperate, concluding: "The point is that the same behavior of keeping your distance also quickens traffic and also prevents accidents."

The questionnaires' analysis show that most but not all students advanced their understanding of traffic for at least one dimension, mainly regarding complex-systems related issues of decentralization, distributions of causes and speed, see Table 1. We see a shift from global "I think the jam is caused from merging a number of lanes …" to emergent-decentralized "Cars slow down and speed up during a traffic jam. I think this happens because not all the drivers drive at the same speed and that causes everybody to slow down, which causes the jam." In the conference, data-logging and "self-aggressive driving" results will be presented.

Significance

Results support the claim for open-ended learning that is supported by a constraining environment's responsiveness and collaboration, and suggest ways to enhance such learning in future experiments. This work aims to heighten safety in driving and create a culture where congestion works optimally.

Tools, Problem Spaces, and Epistemic Games

Michelle Wilkerson & Brian Gravel
Tufts University

Objective

Inquiry-based pedagogies are criticized for involving unstructured searches for solutions. We examine the role that technology-mediated tools can play as scaffolds that allow students to "cognitively manipulate information in ways that are consistent with a learning goal" (Kirschner, Sweller, & Clark, 2006, p. 77). Such tools, we argue, can support particular *epistemic games* (Collins & Ferguson, 1993) that allow learners to organize, compare, and manipulate information and solutions in ways consistent with learning goals.

Theoretical Approach

Discovery-based learning environments engage learners in creating solutions to problems through inquiry and reflection. Often, the construction of digital artifacts is a part of this approach (e.g. Stratford et al., 1998; Van Joolingen & de Jong, 2007; Wilensky & Reisman, 2006). Implicit in these designs is the expectation that technological infrastructures can help learners organize and manipulate relevant subsets of information.

To explore this expectation, we leverage Collins and Ferguson's (1993) notions of *epistemic forms* and *epistemic games*. Epistemic forms are representations (like lists, tables, or diagrams) that allow practitioners to organize and share knowledge, reveal new directions for inquiry, and compare and contrast ideas. *Epistemic games* are the strategies used to populate these forms.

We conjecture that the degree to which learners achieve relevant learning goals and generate problem solutions is related to the alignment between their engagement in certain *epistemic games*, and the *epistemic forms* they have available.

Methods

Student groups created animations and simulations during a two-weeks long classroom activity to explain evaporation and condensation (Wilkerson-Jerde, Gravel & Macrander, 2015a). Our goal was for students to "discover" the particulate and random nature of these phenomena through iterative testing and refinement of these digital artifacts.

We analyze the degree to which students played epistemic games appropriate for the simulation environment - that is, whether they described evaporation in terms of the objects, behaviors, and interactions involved. We also analyze whether students' conversations reflected desired learning goals: mechanistic reasoning (e.g., Russ et al, 2008), and engagement with the random and particulate nature of matter (e.g., Johnstone, 1991).

Figure 12. Group 1 described evaporation in terms of “phases”, each corresponding to a “scene”

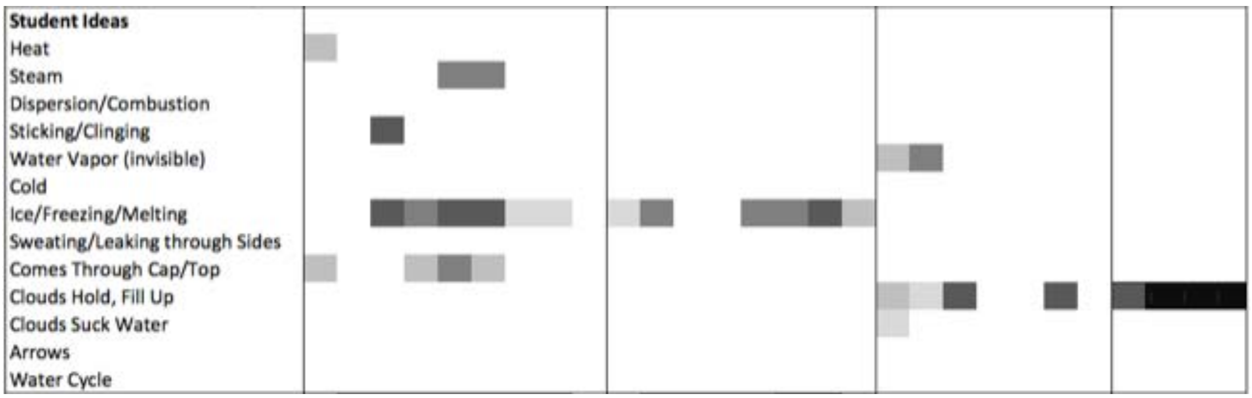


Figure 13. Group 1 did not engage frequently in deep conversation about mechanistic aspects of evaporation (black shading), and did not integrate many ideas in their model or conversation.

Evidence & Results

Here we present content analyses of two focal groups, one successful and one unsuccessful. In the full paper, we will feature all participating student groups.

Group 1 did not engage in the “epistemic game” needed to leverage SiMSAM as an epistemic form (Figure 12). They described evaporation in terms of “phases”, and did not engage deeply in discussion of mechanism (Figure 13); likely drawing on prior school lessons that describe phases of the water cycle.

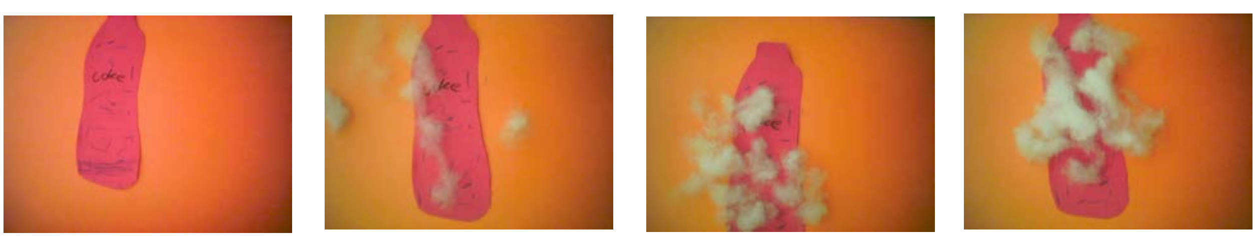


Figure 14. Group 1 described evaporation in terms of “phases”, each corresponding to a “scene”.

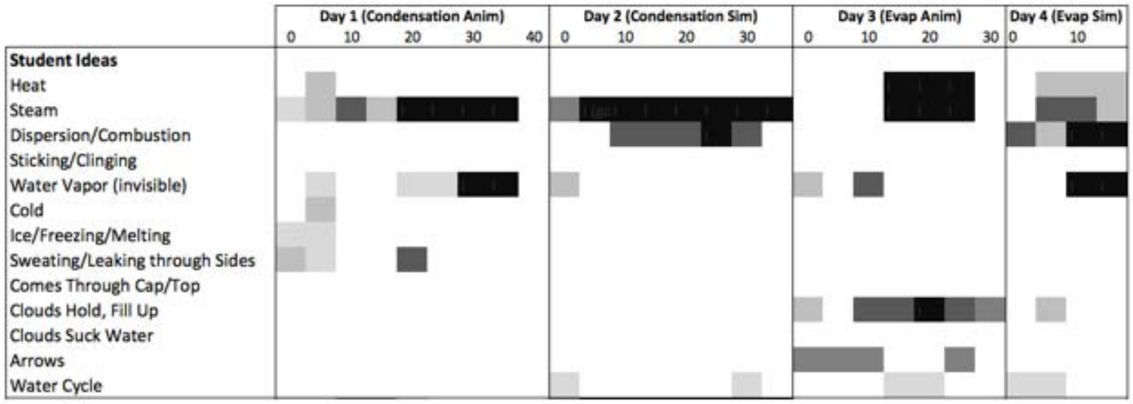


Figure 15. Group 2 frequently engaged in deep conversation about mechanistic aspects of evaporation (black shading), and integrated many ideas into their model and conversation.

Group 2 did engage in the "epistemic game" of identifying objects, behaviors, and interactions. They described evaporation in terms of "vapor" that touches cold surfaces (Figure 14), and engaged deeply in discussion of mechanism and related learning goals (Figure 15); citing experiences with steam where visible particles "combust".

Significance
Examining discovery through the lens of epistemic forms and games can help explain the effectiveness of such interventions, and inform the design of new supports for technology-mediated inquiry environments.

Making "Direct Instruction" and "Discovery Learning" Play Along: Restoring the Historical Educational Role of Practice

Dragan Trninic & Dor Abrahamson

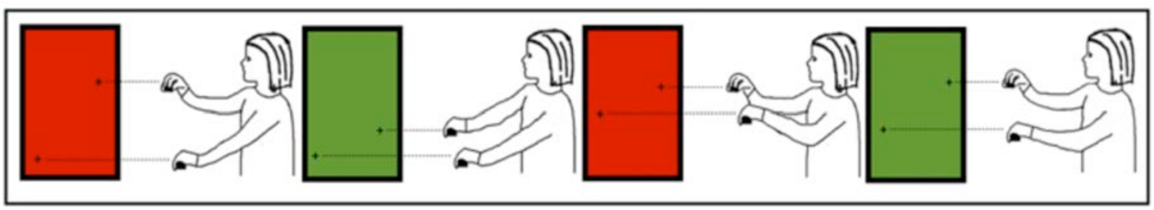
Objective
Dewey (1916) remarked that learning naturally results from doing. We concur and turn our attention to the role of *practice*. Specifically, heeding von Glaserfeld's (1983) call to investigate the pedagogical approaches in the physical disciplines, we first turn our attention to how practice and learning are conceived in certain Chinese martial arts, then attend to parallels in our own design.

Theoretical Approach
Drawing on a multiyear ethnographic study of taiji and yiquan instruction, we outline a pedagogical approach centered on learning from doing. This approach, we offer, does away with the oft-heard tension of "direct instruction" versus "discovery." Namely, students are directly instructed on the *practice*, even as *discovery* is explicitly understood as the purpose of practice.

In making sense of this pedagogical approach, we turn to the work of Soviet neurophysiologist Nikolai Bernstein (1996). Bernstein provides an original and pragmatic framework, wherein teachers provide what he labels *outer* aspects of knowing—i.e., hands go there, elbows here…—yet *inner* aspects of knowing *must be experientially felt* and thus arrive only as personal discovery.

> The fact that the 'secrets' of swimming or cycling *are not in some special body movements but in special sensations and corrections* explains why these secrets are impossible to teach by demonstration" (p. 187).

On this account we agree with Freudenthal, who famously remarked that learning mathematics is like learning to swim (1971). Inspired by this perspective, we strive to design contemporary mathematics learning environments.



a. *b.* *c.* *d.*

*Figure 16***.** The Mathematical Imagery Trainer for Proportion (MIT-P) set at a 1:2 ratio, so that the favorable sensory stimulus (a green background) is activated only when the right hand is twice as high along the monitor as the left hand. This figure sketches out our Grade 4 – 6 study participants' paradigmatic interaction sequence toward discovering an effective operatory scheme: (a) while exploring, the student first positions the hands incorrectly (red feedback); (b) stumbles upon a correct position (green); (c) raises hands maintaining a fixed interval between them (red); and (d) corrects position (green). Compare 1b and 1d to note the different vertical intervals between the virtual objects.

Methods & Design

Next, we present a contemporary mathematics-education motion-sensor learning environment. In our target design, students literally inscribe mathematical proportion with their hand movements (see, e.g., Abrahamson & Trninic, 2015; see Figure 16). Specifically, students engage an educational activity they perceive as a game, yet the goal of which—unbeknownst to them—is to move their hands at proportional rates from some given baseline (e.g., the table). Once students become proficient at this practice, they begin using standard mathematical notations (e.g., numerals, grids) to reflect on and enhance their actions. We draw on microgenetic analysis of video data as well as records of participants' gaze (i.e., eye tracking; see Figure 17) to illustrate how students in our study discover and coordinate various meanings of proportion *through* their practice.

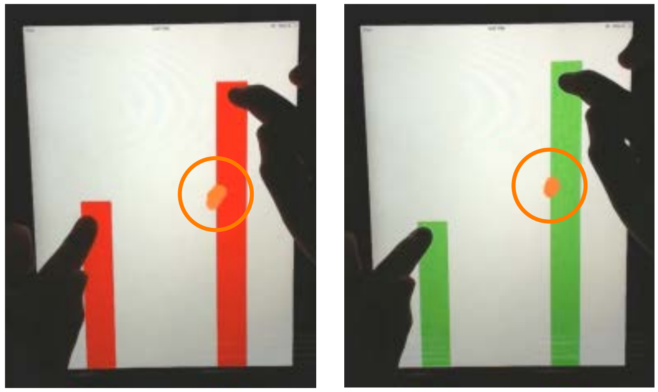


Figure 17. Two temporally consecutive frames of gaze locations before and after finding the next green. The orange dot (highlighted for visibility by an orange circle) indicates the fixation of the eye gaze. The gaze lies not on any shape contour but on an unmarked location half-way up the right-hand bar, and the hands then adjust—lowering the left hand, raising the right hand—to effect the 1:2 ratio. Our analysis highlights how the composite aspects of the ratio (left goes up by 1, right by 2) are visually coordinated into a whole (1:2).

Significance

In both traditional Chinese martial arts classes and our own design for mathematical proportion, the student is *first* given a practice and then, *through this practice*, discovers disciplinary meanings. While we cannot directly tell "the secrets" (Freudenthal, 1971) of our disciplines, we can *and should* provide students, as clearly as possible, with the sorts of practices that lead to meaningful personal discovery. Hence, "direct instruction" and "discovery" are not opposite (also see Taber, 2010) but complementary. Conceived thus, "direct instruction" is not used to *tell* the disciplinary meanings (because this would be a waste of time), but rather to instruct practices that lead to discovery and meaning-making, that is to say, that lead to learning.

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